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Propagation of Waves in Random Rotating Infinite Magneto-Thermo-Visco-Elastic Medium

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Abstract—The problem of wave propagation in an interacting random infinite magneto-thermo-visco-elastic medium has been studied formulating a generalised theory of thermoelasticity recently proposed by Noda, Furuk-Awa and Ashida [1] that combines both the generalised theories developed by Lord and Shulman [2] as well as by Green and Lindsay [3]. The perturbation technique relevant to stochastic differential equations has been employed to obtain the relation connecting displacement amplitudes of waves propagating in the interacting media. The appropriate Green's tensor essential for the discussion has been obtained in the course of analysis. A more general coupled dispersion relation for longitudinal and transverse waves has been deduced to determine the effect of visco-elasticity, relaxation times, and rotation on the phase velocity of the coupled waves. The equations have been analysed for a particular form of thermo-mechanical coupling and autocorrelation function to show that the effect (of the order of ϵ^2 only) of the thermal field is to attenuate the longitudinal type waves and to alter the phase-speed depending upon the values of the visco-elastic parameters, relaxation times, and rotation. Cases of low and high frequencies have also been studied, and numerical calculations have been done to determine the effect of visco-elastic parameters, relaxation times, rotation, and thermoelastic coupling on the phase velocity and attenuation coefficient of the waves.

1. INTRODUCTION

In recent years, considerable interest has been shown in the study of plane thermoelastic, magneto-thermoelastic, and magneto-thermo-visco-elastic wave propagation in an infinite nonrandom or random and nonrotating or rotating medium by many authors [4–7] following the classical theory of thermoelasticity which is based on Fourier's law of heat conduction. This law predicts an infinite speed of propagation of heat which is physically absurd, and as a result, many new theories have been proposed to eliminate this absurdity. Lord and Shulman (L-S) [2] proposed a modified version of Fourier's law and deduced a theory of thermoelasticity known as the generalised theory of thermoelasticity. This theory with a thermal relaxation time has been used with purpose and profit by many authors [8–17] to study the effect of thermoelastic, magneto-thermoelastic, and magneto-thermo-visco-elastic plane wave propagation in an infinite nonrotating or rotating medium.

Another theory of thermoelasticity has been proposed by Green and Lindsay (G-L) [3] which has certain special features in contrast with the previous theory proposed by L-S. In this theory of Green and Lindsay, Fourier's law of heat conduction remains unchanged, whereas the classical energy equation and the stress-strain temperature relations are modified. Two constitutive constants α and α^* having the dimensions of time appear in the governing equations in place

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of one relaxation time τ in Lord-Shulman's theory. Using the G-L theory, Agarwal [10,12] studied thermoelastic and magneto-thermoelastic plane wave propagation in an infinite nonrotating medium. Roychoudhuri [13,14] studied the propagation of harmonically time-dependent plane thermoelastic waves in an infinite rotating medium in the context of linearised theory of G-L.

In the present paper, however, the problem of wave propagation in a random rotating magneto-thermo-visco-elastic medium has been investigated by formulating a generalised theory of thermoelasticity [1] that combines both the theories developed in [2,3]. A random or stochastic medium which differs slightly from a homogeneous medium is a mathematical model of a complex medium and consists of a family of media together with a probability distribution which specifies the probabilities of the various members of the family. Thus, wave propagation in a random medium refers to propagation in each member of the family of media, together with the probability of each member. Furthermore, it appears that little attention has been paid to the study of propagation of plane thermoelastic or thermo-visco-elastic waves in random rotating medium.

That is why the propagation of small amplitude plane complex elastic and electromagnetic waves in a such a system has been studied. The resulting magnetic field is $\vec{H}_0 + \vec{h}$, where $\vec{H}_0$ is the external magnetic field and $\vec{h}$, the small perturbation in the magnetic field. The dispersion relation is evaluated, neglecting small terms of second and higher order such as $\frac{\partial \vec{u}}{\partial t} \times \vec{h}$, $\operatorname{curl} \vec{h} \times \vec{h}$, $\nabla(1/\sigma) \times \operatorname{curl} \vec{h}$, etc., when the conductivity and other material constants undergo small random variations.

A more general dispersion equation is obtained to determine the effects of relaxation, randomness, and rotation on the phase velocity of the waves. The mathematical study of elastic and electromagnetic wave propagation in random materials was carried out by Keller [18] and by Karal and Keller [19]. Their smooth perturbation method has been employed for the solution of the resulting stochastic differential equations. This method consists in deriving an integro-differential equation for the mean field correct up to the second order in ϵ, where ϵ measures the smallness of the scale of fluctuation about a homogeneous medium. The field equations are put in the form

$$LV = f, \tag{1.1}$$

where L is a random linear operator, V the field quantity, and f the nonrandom source term. Assuming that

$$L = L_0 + \epsilon L_1 + \epsilon^2 L_2, \tag{1.2}$$

Keller [18] has shown that the mean field quantity $\langle V \rangle$ satisfies the equation

$$\left[L_0 + \epsilon\langle L_1\rangle + \epsilon^2\langle L_2\rangle + \epsilon^2\left(\langle L_1\rangle L_0^{-1}\langle L_1\rangle - \langle L_1 L_0^{-1} L_1\rangle\right)\right]\langle V\rangle = f, \tag{1.3}$$

where L_0^{-1} is obtained [20] by solving

$$L_0 G_{ij}\left(\vec{x}, \vec{x}'\right) = \delta\left(\vec{x}, \vec{x}'\right)\delta_{ij}, \tag{1.4}$$

where G_{ij} is the appropriate Green's tensor for L_0.

In the present investigation, equation (1.3) has been used as the equation governing the mean field quantity, a set of eight vectors, one more than those used in the previously discussed problem of nonrotating medium [21]. The visco-elastic medium has been assumed to be weakly thermal and also weakly conducting. By a weakly thermal and weakly conducting medium, we mean that the thermo-mechanical coupling parameter and the conductivity are random functions, proportional to ϵ, with nonzero mean values. A constant magnetic field $\mathbf{H_0}$ is assumed. The associated Green's tensor (for magneto-visco-elastic medium in generalised thermoelasticity) has already been evaluated by Bera [20]. The relation connecting the displacement amplitudes is presented and it contains a large number of terms up to the order ϵ^2. The thermal effect is not discernible up to ϵ-order terms, thereby showing that magnetic effect is more important so far as wave propagation in the interacting field is concerned. All cross-correlation functions between thermal and magnetic parameters disappear in the dispersion equation. Finally, in order to study a particular case, all the correlation functions except the thermal coupling correlation function are assumed to be zero. The various terms of the reduced equation are then interpreted. Terms to the order ϵ^2 representing magnetic effect still exist as $\langle L_2 \rangle \neq 0$ in this case. Furthermore, terms representing thermal effects agree with those obtained by Chow [4] and Bhattacharyya [6], if $(t_0, t_1, \Omega) \to 0$.

Further simplification is brought about by neglecting ϵ^2 order terms representing the effect of magnetic field. This may be justified in view of the fact that ϵ-order effect is already present. As a result, the dispersion equation splits into two equations. The k_c and k_s type waves are coupled in the one containing magnetic and thermal terms. The second equation represents k_s type uncoupled waves propagating with amplitude in the z-direction (i.e., perpendicular to magnetic field and direction of propagation), the waves being affected by magnetic field alone. These equations can be analysed by using the method adopted by Chow [4]. The coupled dispersion equation has, in fact, been analysed for longitudinal type waves for a Voigt-type visco-elastic solid. The effect of the various perturbation terms appearing in the dispersion equation has been examined for high and low frequency fields.

If λ, μ represent Lamé elastic moduli, all the results are valid for wave propagation in a generalised magneto-thermoelastic medium.

Finally, it may be mentioned here that if the relaxation parameters t_0 and t_1 and the rotation parameter Ω in the present analysis in respect of generalised thermoelasticity are allowed to tend to zero, the results obtained here are found to be in excellent agreement with those of Bhattacharyya [6] in classical thermoelasticity after some corrections.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

An infinite isotropic, homogeneous (thermally as well as electrically) conducting random magneto-thermo-visco-elastic medium with density ρ at uniform initial temperature θ_0 has been considered. The medium is rotating with an angular velocity $\Omega = \Omega \mathbf{n}$, where $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference has two additional terms:

(i) centripetal acceleration $\Omega \times (\Omega \times \mathbf{u})$ due to the time varying motion only;
(ii) the Coriolis acceleration $2\Omega \times \mathbf{u}$.

Here $\mathbf{u}$ is the dynamic displacement vector measured from a steady state deformed position and supposed to be small. Let $\mathbf{H}$ be the magnetic vector and $\mathbf{j}$ the current density vector of the electromagnetic field.

Neglecting the small quantity $\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{h}$, $\nabla(1/\sigma) \times \operatorname{curl} \mathbf{h}$, and $(\operatorname{curl} h) \times h$, the magnetic perturbation field equation is obtained in the form

$$
\begin{aligned}
-\nu \frac{\partial \mathbf{h}}{\partial t} = \frac{1}{\sigma}\left[\nabla(\operatorname{div} \mathbf{h}) - \nabla^2 \mathbf{h}\right] + \nu \left[\mathbf{H}\left(\nabla \cdot \frac{\partial \mathbf{u}}{\partial t}\right) - (H_0 \cdot \nabla) \frac{\partial \mathbf{u}}{\partial t}\right] \\
+ \left(\nabla \nu \cdot \frac{\partial \mathbf{u}}{\partial t}\right)\left[\mathbf{H}_0 - (\nabla \nu \cdot \mathbf{H}_0) \frac{\partial \mathbf{u}}{\partial t}\right].
\end{aligned} \tag{2.1}
$$

Then the displacement equation of motion in the magneto-thermo-visco-elastic medium with rotating frame of reference can be written as

$$
\begin{aligned}
\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla(\nabla \cdot \mathbf{u}) + \nabla \times (\nabla \cdot \mathbf{u}) + \nabla \mu \times (\nabla \times \mathbf{u}) \\
+ 2(\nabla \mu \cdot \nabla) \mathbf{u} - \nu \mathbf{H}_0 \times (\nabla \times \mathbf{h}) - \nabla \left\{ m \left(\theta + t_1 \dot{\theta} \right) \right\} \\
= \rho \left[\ddot{u} + \Omega \times (\Omega \times \mathbf{u}) + 2\Omega \times \dot{\mathbf{u}} \right],
\end{aligned} \tag{2.2}
$$

where

$$\lambda = \frac{1}{3}(\gamma_\nu - \gamma_s), \qquad \mu = \frac{1}{2}\gamma_s,$$

γ_s and γ_ν being the deviatoric and dilatational complex moduli, respectively [22].

Assuming that

$$(\mathbf{u}, \mathbf{h}, \theta) = (\bar{\mathbf{u}}, \bar{\mathbf{h}}, \bar{\theta})e^{-i\omega t}, \tag{2.3}$$

equation (3.1) reduces to

$$\begin{aligned} i\nu\omega\bar{h} = -\frac{1}{\sigma}\left[\nabla\left\{\frac{1}{\nu}(\nabla\nu)\cdot\bar{\mathbf{h}}\right\} + \nabla^2\bar{\mathbf{h}}\right] - i\omega\nu\,[H_0(\nabla\cdot\bar{\mathbf{u}}) \\ - (H_0\cdot\nabla)\bar{\mathbf{u}}] - i\omega(\nabla\nu\cdot\bar{\mathbf{u}})\mathbf{H}_0, \end{aligned} \tag{2.4}$$

with $\nabla\nu\cdot H_0 = 0$, which indicates that there is no variation of ν in the direction of the unperturbed magnetic field. Then using (2.3), the energy equation proposed in [1] can be written as

$$-\gamma\omega\left(\bar{\theta}i + t_0\omega\bar{\theta}\right) = \nabla\cdot\left(\eta\nabla\bar{\theta}\right) + \omega m\theta_0\nabla\cdot(i\bar{\mathbf{u}} + \delta_{lk}t_0\omega\bar{\mathbf{u}}). \tag{2.5}$$

For the L-S theory, $t_1 = 0$, $\delta_{lk} = 1$, and for the G-L theory, $t_1 > 0$ and $\delta_{lk} = 0$ ($k = 1$ for L-S and 2 for G-L theory). The thermal relaxation times t_0 and t_1 satisfy the inequalities [3]

$$t_1 \geq t_0 \geq 0. \tag{2.6}$$

Equation (2.2) reduces to

$$\begin{aligned} &\mu\nabla^2\bar{\mathbf{u}} + (\lambda+\mu)\nabla(\nabla\cdot\bar{\mathbf{u}}) + (\nabla\lambda)(\nabla\cdot\bar{\mathbf{u}}) + (\nabla\mu)\times(\nabla\times\bar{\mathbf{u}}) \\ &\quad + 2\left\{(\nabla\mu)\cdot\nabla)\bar{\mathbf{u}} - \nu\mathbf{H}_0\times(\nabla\times\bar{\mathbf{h}}) - \nabla\{m\left(\bar{\theta} - i\omega t_1\bar{\theta}\right)\right\} \\ &\quad = \rho\left[-\omega^2\bar{\mathbf{u}} + \Omega\times(\Omega\times\bar{\mathbf{u}}) - 2i\omega\Omega\times\bar{\mathbf{u}}\right]. \end{aligned} \tag{2.7}$$

The field equations (2.4), (2.6), and (2.7) can now be put in the form

$$LV = 0, \tag{2.8}$$

where

$$L = \begin{bmatrix} M & P & K \\ N & Q & 0 \\ R & 0 & S \end{bmatrix}, \qquad V = \begin{bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{h}} \\ \bar{\theta} \end{bmatrix}, \tag{2.9}$$

and M, N, P, Q, R, S are determined from the governing equations with

$$\alpha^* = mt_1, \qquad \beta^* = mt_0, \qquad \delta^* = \gamma t_0, \tag{2.10}$$

assuming that

$$\begin{aligned} (\lambda, \mu, \nu, \gamma, \alpha^*, \beta^*, \delta^*, \eta, \rho) &= (\lambda_0, \mu_0, \nu_0, \gamma_0, \alpha_0^*, \beta_0^*, \delta_0^*, \eta_0, \rho_0) \\ &\quad + \varepsilon(\lambda_1, \mu_1, \nu_1, \gamma_1, \alpha_1^*, \beta_1^*, \delta_1^*, \eta_1, \rho_1) \end{aligned} \tag{2.11}$$

and

$$\sigma = \epsilon\sigma_1, \qquad m = \epsilon m_1, \tag{2.12}$$

such that

$$\begin{aligned} \langle\sigma_1\rangle = \sigma_1^0 \neq 0, \qquad \langle m_1\rangle = m_1^0 \neq 0, \\ \langle\alpha_1^*\rangle = (\alpha_1^*)^0 \neq 0, \qquad \langle\beta_1\rangle = (\beta_1^*)^0 \neq 0, \end{aligned} \tag{2.13}$$

and

$$\langle\lambda_1\rangle = \langle\mu_1\rangle = \langle\nu_1\rangle = \langle\gamma_1\rangle = \langle\delta_1^*\rangle = \langle\eta_1\rangle = \langle\rho_1\rangle = 0. \tag{2.14}$$

By virtue of (2.12), (2.13), and (2.14), we obtain that

$$\begin{aligned}
\langle M_1\rangle &= 0 = \langle P_1\rangle,\\
\langle K_1\rangle &= -\left\{m_1^0(1-i\omega t_1)\right\}\nabla,\\
\langle N_1\rangle &= i\sigma_1^0\omega\nu_0\left[(\mathbf{H}_0\cdot\nabla)-\mathbf{H}_0(\nabla\cdot)\right],\\
\langle Q_1\rangle &= -[i\omega\nu_0\sigma_1^0],\\
\langle R_1\rangle &= (i+\delta_{lk}\omega t_0)\omega\theta_0 m_1^0(\nabla\cdot),\\
\langle S_1\rangle &= 0,
\end{aligned} \tag{2.15}$$

$$\begin{aligned}
\langle N_2\rangle &= i\omega\left[\langle\sigma_1\nu_1\rangle\left\{(\mathbf{H}_0\cdot\nabla)-\mathbf{H}_0(\nabla\cdot)\right\}-\mathbf{H}_0\left\{\langle\sigma_1(\nabla\nu_1)\rangle\cdot\right\}\right],\\
\langle Q_2\rangle &= \frac{1}{\nu_0^2}\left[\nabla\left\{\langle\nu_1(\nabla\nu_1)\cdot\rangle\right\}\right]-i\omega\langle\nu_1\sigma_1\rangle.
\end{aligned} \tag{2.16}$$

Thus,

$$\langle L_1\rangle = \begin{bmatrix} 0 & 0 & \langle K_1\rangle \\ \langle N_1\rangle & \langle Q_1\rangle & 0 \\ \langle R_1\rangle & 0 & 0 \end{bmatrix} \neq 0, \qquad \langle L_2\rangle = \begin{bmatrix} 0 & 0 & 0 \\ \langle N_2\rangle & \langle Q_2\rangle & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq 0. \tag{2.17}$$

3. SOLUTION OF THE PROBLEM

Let the mean field quantity

$$\langle v(x)\rangle = \begin{bmatrix} A \\ B \\ D \end{bmatrix} e^{ik\cdot x} \tag{3.1}$$

satisfy the integro-differential equation

$$\left[L_0 + \epsilon\langle L_1\rangle + \epsilon^2\langle L_2\rangle + \epsilon^2\left(\langle L_1\rangle L_0^{-1}\langle L_1\rangle - \langle L_1 L_0^{-1} L_1\rangle\right)\right]\langle v(\mathbf{x})\rangle = 0. \tag{3.2}$$

The appropriate Green's tensor corresponding to L_0^{-1} is

$$G = \begin{bmatrix} G_0 & G_1 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}. \tag{3.3}$$

The components of G_0, G_1, G_2 have already been evaluated by Bera [20] in the generalised theory of thermoelasticity. Also, $G_3(r)$, being the solution of the equation

$$\left(\nabla^2 + \frac{i\omega\gamma_0 + \delta_0^*\omega^2}{\eta_0}\right) G_3 = \delta(x - x'), \tag{3.4}$$

is given by

$$G_3(r) = -\frac{1}{4\pi r}\cdot e^{i\beta r}, \tag{3.5}$$

where

$$\beta = \left(\frac{\gamma_0\omega}{\eta_0}\right)^{1/2}\left(1+\frac{\omega^2\delta_0^{*2}}{\gamma_0^2}\right)^{1/4}\cdot e^{i\theta'/2}, \qquad \theta' = \tan^{-1}\left(\frac{1}{\omega t_0}\right). \tag{3.6}$$

As $t_0 \to 0$,

$$\theta' \to \frac{\pi}{2} \qquad \text{and} \qquad \beta\sqrt{\frac{\omega\gamma_0}{2\eta_0}}\,(1+i). \tag{3.7}$$

The result (3.7) exactly coincides with that obtained earlier in [6].

Substitution of (3.1) into (3.2) yields the following three equations:

$$
\begin{gathered}
(M_0' + P_0') - m_1^0(1 - i\omega t_1)iDk\epsilon \\
+ \left[(1 - i\omega t_1)(i + \delta_{lk}t_0\omega)\cdot\omega\theta_0\left(-m_1^0\right)^2(A\cdot k)k\int G_3(r)e^{-ik\cdot r}\,dr\right]\epsilon^2 \\
+ \epsilon^2\int [f_1A + f_2B + f_3D]e^{-ik\cdot r}\,dr,
\end{gathered} \tag{3.8}
$$

where

$$
\begin{aligned}
M_0' &= -(\lambda_0 + \mu_0)(k\cdot A)k + \left(\rho_0\omega^2 - \mu_0 k^2\right)A \\
&\quad + \rho_0\left\{A\Omega^2 - \Omega(\Omega\cdot A) + 2i\omega(\Omega\times A)\right\}, \\
P_0' &= -i\nu_0\mathbf{H}_0\times(k\times B),
\end{aligned} \tag{3.9}
$$

and

$$
\begin{aligned}
e^{ik\cdot x'}(f_1A) &\equiv \langle M_1G_0M_1' + M_1G_1N_1' + P_1G_2N_1' + K_1G_3R_1'\rangle\left(Ae^{ik\cdot x'}\right), \\
e^{ik\cdot x'}(f_2B) &\equiv \langle M_1G_0P_1' + M_1G_1Q_1' + P_1G_2Q_1'\rangle\left(Be^{ik\cdot x'}\right), \\
e^{ik\cdot x'}(f_3D) &\equiv \langle M_1G_0K_1' + K_1G_3S_1'\rangle\left(De^{ik\cdot x'}\right),
\end{aligned} \tag{3.10}
$$

and two other similar equations.

Retaining terms up to the order ϵ^2 in the last two equations, we obtain that

$$
B = \frac{1}{k^2}\left(1 + \frac{1\omega\nu_0\sigma_1^0\epsilon}{k^2}\right)\left[\epsilon\sigma_1^0\omega\nu_0\{k\times(A\times H_0)\} + \epsilon^2\psi\right], \tag{3.11}
$$

where

$$
\begin{aligned}
\psi &= \omega[\langle\sigma_1\nu_1\rangle\{k\times(A\times H_0)\} - iH_0\langle\sigma_1\nabla\nu_1\cdot A\rangle] \\
&\quad - \left[\left(\sigma_1^0\omega\nu_0\right)^2\{k\times(H_0\times A)\}\int\{G_1(r)(H_0\cdot k) + iG_2(r)\}e^{-ik\cdot r}\,dr\right. \\
&\quad \left. + \int G_1Ae^{-ik\cdot r}\,dr\right],
\end{aligned} \tag{3.12}
$$

and

$$
D = \frac{\epsilon\omega\theta_0 m_1^0(A\cdot k) - \epsilon^2\int h_1Ae^{-ik\cdot r}\,dr}{i\omega\gamma_0 - \eta_0k^2}. \tag{3.13}
$$

Inserting B and D in equation (3.8), we obtain that

$$
\begin{gathered}
M_0' - i\nu_0H_0\times\left[k\times\left\{\frac{\epsilon\omega\gamma_0\sigma_1^0}{k^2}(k\times(A\times H_0)) + \epsilon^2\psi + i\left(\omega\nu_0\sigma_1^0\right)^2\frac{\epsilon^2}{k^2}(k\times(A\times H_0))\right\}\right] \\
- \epsilon^2\left(m_1^0\right)^2(1 - i\omega t_1)i\mathbf{k}\cdot\frac{\omega\theta_0(A\cdot k)}{i\omega\gamma_0 - \eta_0k^2} - \epsilon^2\left(m_1^0\right)^2(1 - i\omega t_1)\cdot\omega\theta_0\cdot ik(A\cdot k) \\
\cdot\int G_3(r)e^{-ik\cdot r}\,dr + \epsilon^2\int f_1Ae^{-ik\cdot r}\,dr = 0.
\end{gathered} \tag{3.14}
$$

Equation (3.14) represents the relation connecting displacement amplitudes for waves propagating in the interacting rotating random magneto-thermo-visco-elastic medium in respect of the generalised theory of thermoelasticity. The term M_0' contains the rotating parameter Ω. If λ, μ represent Lamé parameters, then this equation represents the corresponding equation for magneto-thermoelastic waves. Furthermore, if $\Omega = 0$ and $t_0 = t_1 = 0$, then this equation exactly coincides with the corresponding equation in the classical theory of elasticity [6]. However, it may be noted that if the terms up to the order ϵ^2 are retained, this equation does not include any term involving cross-correlation function between magnetic and thermal parameters.

If the terms to the order ϵ only are retained, equation (3.14) reduces to

$$\begin{aligned}&\left[-(\lambda_0+\mu_0)(k\cdot A)k+\left(\rho_0\omega^2-\mu_0k^2\right)\vec{A}+\rho_0\left\{\Omega^2\vec{A}-\left(\vec{\Omega}\cdot\vec{A}\right)\vec{\Omega}+2i\omega\left(\vec{\Omega}\times\vec{A}\right)\right\}\right]k^2\\&-i\omega\nu_0^2\sigma_1^0\in\left[(k\cdot H_0)\left\{(H_0\cdot A)k-(H_0\cdot k)A\right\}-(k\cdot A)\cdot\left\{H_0^2k-(H_0\cdot k)H_0\right\}\right]=0.\end{aligned}\tag{3.15}$$

The terms representing the effect of the thermal field do not appear either in the classical theory [6] or in the generalised theory of elasticity [20], while the effect of the magnetic field is discernible to the first order of ϵ. This indicates that in an interacting field of the type described here, the effect of the magnetic field is stronger than that of the thermal field. To study the effect of the presence of a thermal field, we assume for equation (3.14) that

$$R_{mm}(r)=\langle m_1(x)m_1(x')\rangle\neq0,\tag{3.16}$$

and all other correlation functions vanish. Furthermore, in this case we choose to neglect terms representing the effect of magnetic field to the order of ϵ^2. This is so because the magnetic effects to the order of ϵ are already at hand. The resulting equation is

$$\begin{aligned}&\left[-(\lambda_0+\mu_0)(k\cdot A)k+\left(\rho_0\omega^2-\mu_0k^2\right)A+\rho_0\left\{\Omega^2A-(\Omega\cdot A)\Omega+2i\omega(\Omega\times A)\right\}\right]k^2\\&\quad-i\omega\nu_0^2\sigma_1^0\in\left[(k\cdot H_0)\{(H_0\cdot A)k-(H_0\cdot k)A\}-(k\cdot A)\left\{H_0^2k-(H_0\cdot k)H_0\right\}\right]\\&-\frac{i\omega\theta_0\left(m_1^0\right)^2(1-i\omega t_1)}{i\omega\gamma_0-\eta_0K^2}\cdot\epsilon^2\cdot(A\cdot k)k+\epsilon^2\cdot\left(m_1^0\right)^2(1-i\omega t_1)\omega\theta_0(i+\delta_{lk}t_0\omega)k(A\cdot k)\\&\qquad\int G_3(r)e^{-ik\cdot r}\,dr-\epsilon^2\int\langle k_1G_3R_1'\rangle\,Ae^{-ik\cdot r}\,dr=0,\end{aligned}\tag{3.17}$$

where

$$\int\langle k_1G_3R_1'\rangle\,Ae^{-ik\cdot r}\,dr=(1-i\omega t_1)(i+\delta_{lk}\omega t_0)\omega\theta_0k(k\cdot A)\cdot\int R_{mm}(r)G_3(r)e^{-ik\cdot r}\,dr.\tag{3.18}$$

Without loss of generality, we may take

$$k=(k,0,0),\qquad H_0=(H_{01},H_{02},0),\qquad A=(A_1,A_2,A_3),\quad\text{and}\quad\Omega=(0,0,\Omega).\tag{3.19}$$

Equation (3.17) splits into the following three equations after taking care of (3.18) and (3.19):

$$\begin{aligned}&\left[(\lambda_0+2\mu_0)k^2-\rho_0(\omega^2+\Omega^2)\right]A_1+2i\omega\Omega\rho_0A_2+i\omega\nu_0^2\sigma_1^0H_{02}(H_{01}A_2-H_{02}A_1)\\&\qquad+\frac{i\omega\theta_0\left(m_1^0\right)^2(1-i\omega t_1)}{i\omega\gamma_0-\eta_0k^2}A_1k^2\epsilon^2+\epsilon^2(1-i\omega t_1)k^2\\&\quad\cdot(i+\sigma_{lk}\omega t_0)\omega\theta_0A_1\int\left[R_{mm}(r)-\left(m_1^0\right)^2\right]G_3(r)e^{-ik\cdot r}\,dr=0,\end{aligned}\tag{3.20}$$

$$\left[\rho_0(\omega^2+\Omega^2)-\mu_0k^2\right]A_2+2i\omega\Omega\rho_0A_1-\epsilon i\omega\sigma_1^0\nu_0^2H_{01}[H_{02}A_1-H_{01}A_2]=0,\tag{3.21}$$

and

$$\left[\mu_0k^2-\rho_0\omega^2-\epsilon i\omega\sigma_1^0\nu_0^2H_{01}^2\right]A_3=0.\tag{3.22}$$

Eliminating A_1, A_2 from (3.20) and (3.21), using (3.5) and the relation

$$\iiint F(r)e^{ik\cdot r}\,dr=\frac{4\pi}{k}\int_0^\infty rF(r)\sin kr\,dr\tag{3.23}$$

and keeping the terms up to the order of ϵ^2, one obtains that

$$\begin{aligned}&\left[(\lambda_0+2\mu_0)k^2-\rho_0\left(\omega^2+\Omega^2\right)\right]\left[\mu_0k^2-\rho_0\left(\omega^2+\Omega^2\right)\right]\\&-\epsilon i\omega\sigma_1^0\nu_0^2\left[H_{01}^2\{(\lambda_0+2\mu_0)k^2-\rho_0(\omega^2+\Omega^2)\}+H_{02}^2\left\{\mu_0k^2-\rho_0\left(\omega^2+\Omega^2\right)\right\}\right]\\&+\epsilon^2\left[\mu_0k^2-\rho_0\left(\omega^2+\Omega^2\right)\right]\cdot\left[D_3(k)+D_4(k)\right]-4\omega^2\Omega^2\rho_0^2=0,\end{aligned}\tag{3.24}$$

where

$$D_3(k) = -\frac{i\left(m_1^0\right)^2 k^2 \left(i\omega\gamma_0 + \eta_0 k^2\right) \omega\theta_0(1 - i\omega t_1)}{(\omega^2\gamma_0^2 + \eta_0^2 k^4)}, \tag{3.25}$$

and

$$D_4(k) = (1 - i\omega t_1)(i + \delta_{lk}\omega t_0)\omega\theta_0 k \int_0^\infty \left\{R_{mm}(r) - \left(m_1^0\right)^2\right\} e^{i\beta r} \sin kr\, dr. \tag{3.26}$$

Therefore, equation (3.24) is the coupled dispersion equation for longitudinal and transverse waves propagating in the medium under consideration. Equation (3.22) gives the dispersion equation for the transverse type of waves propagating with a single velocity independent of thermal effect and rotation. In both the cases, λ_0 and μ_0 represent complex visco-elastic moduli. When λ_0 and μ_0 denote elastic Lamé parameters and $(t_0, t_1, \Omega) \to 0$, the expressions for D_3 and D_4 coincide with those obtained by Chow [4]. Furthermore, if $(t_0, t_1, \Omega) \to 0$, equation (3.24) along with (3.25) and (3.26) exactly coincide with the corresponding results in the classical theory obtained in [6]. Finally, if $\Omega \to 0$, then the above results can easily be identified with those obtained by Bera [21] following G-L theory [3].

4. ANALYSIS OF THE DISPERSION EQUATION (3.24) IN A PARTICULAR CASE

Let

$$k = k_c - i\delta_c, \qquad k_c^2 = \frac{\rho_0(\omega_0^2 + \Omega^2)}{\lambda_0 + 2\mu_0} \tag{4.1}$$

be a solution of equation (3.24). Substituting (4.1) in (3.24), retaining terms up to the order ϵ^2, and neglecting δ_c^2, since δ_c is small, we obtain, on simplification:

$$\begin{aligned}\delta_c \simeq & -\frac{1}{2}\frac{\omega}{\omega_1} \cdot \frac{\nu_0^2\sigma_1^0 H_{02}^2}{\sqrt{\rho_0(\lambda_0 + 2\mu_0)}}\epsilon - \frac{i\epsilon^2}{2} \cdot \frac{D_3(k_c) + D_4(k_c)}{\omega_1\sqrt{\rho_0(\mu_0 + 2\mu_0)}} \\ & + i\frac{\left(\omega/\omega_1 \cdot \nu_0^2\sigma_1^0 H_{02}\right)^2 \epsilon^2}{2\omega_1\sqrt{\rho_0(\lambda_0 + 2\mu_0)\rho_0(\lambda_0 + \mu_0)}} \cdot \left\{H_{01}^2(\lambda_0 + 2\mu_0) + \mu_0 H_{02}^2\right\} \\ & - \frac{2\rho_0\omega^2\Omega^2 i}{\omega_1^2(\lambda_0 + \mu_0)} \cdot \frac{1}{k_c}\left[1 - \frac{i\epsilon\omega\nu_0^2\sigma_1^0}{\rho_0\omega_1^2(\lambda_0 + \mu_0)}\left\{H_{01}^2(\lambda_0 + 2\mu_0) + \mu_0 H_{02}^2\right\}\right. \\ & \left. + \frac{\mu_0(D_3(k_c) + D_4(k_c))}{\rho_0\omega_1^2(\lambda_0 + \mu_0)} \cdot \epsilon^2\right],\end{aligned} \tag{4.2}$$

where $\omega_1^2 = \omega^2 + \Omega^2$.

If $\Omega \to 0$, $\omega_1 \to \omega$ and putting $\delta_{lk} = 0$, equation (4.2) will exactly coincide with that obtained earlier by Bera [21] using G-L theory.

Since the effect of magnetic field is discernible to ϵ-order, we choose to neglect the third term of equation (4.2) which represents the effect of the magnetic field to the order ϵ^2. Hence, the expression for δ_c becomes approximately

$$\begin{aligned}\delta_c \simeq & -\frac{\epsilon\sigma_1^0\nu_0^2 H_{02}^2}{2\sqrt{\rho_0(\lambda_0 + 2\mu_0)}} \cdot \frac{\omega}{\omega_1} - i\epsilon^2\frac{[(D_3(k_c) + D_4(k_c)]}{2\omega_1\sqrt{\rho_0(\lambda_0 + 2\mu_0)}} \\ & - \frac{2\rho_0\omega^2\Omega^2 i}{\omega_1^2(\lambda_0 + \mu_0)} \cdot \frac{1}{k_c}\left[1 - \frac{i\epsilon\omega\sigma_1^0\nu_0^2}{\rho_0(\lambda_0 + \mu_0)\omega_1^2}\left\{H_{01}^2(\lambda_0 + 2\mu_0) + \mu_0 H_{02}^2\right\}\right. \\ & \left. + \epsilon^2\mu_0 \cdot \frac{(D_3(k_c) + D_4(k_c))}{\rho_0(\lambda_0 + \mu_0)\omega_1^2}\right].\end{aligned} \tag{4.3}$$

If the visco-elastic medium under consideration is of Voigt type, then

$$S_{ij} = 2G(e_{ij} + \theta_1\dot{e}_{ij}),$$

and hence,

$$\lambda_0 = \frac{1}{3}\left(\gamma_\nu^0 - \gamma_s^0\right) = \frac{1}{3}\left[\bar{G}^0 - G^0 - i\omega\left(\bar{G}^0\bar{\theta}_1^0 - G^0\theta_1^0\right)\right]$$

and

$$\mu_0 = \frac{1}{2}\gamma_s^0 = G^0\left(1 - i\omega\theta_1^0\right), \tag{4.4}$$

where G^0, θ_1^0, $\bar{G}^0$, and $\bar{\theta}_1^0$ are the nonrandom parts of G, θ_1, $\bar{G}$, and $\bar{\theta}_1$, respectively. Let

$$(\lambda_0 + 2\mu_0)^{1/2} = \frac{1}{\sqrt{3}}(a + ib), \tag{4.5}$$

where

$$\begin{aligned} a &= \frac{1}{\sqrt{2}}\left[\left(\bar{G}^0 + 5G^0\right) + \left\{\left(\bar{G}^0 + 5G^0\right)^2 + \omega^2\left(\bar{G}^0\bar{\theta}_1^0 + 5G^0\theta_1^0\right)^2\right\}^{1/2}\right]^{1/2}, \\ b &= -\frac{1}{\sqrt{2}}\left[-\left(\bar{G}^0 + 5G^0\right) + \left\{\left(\bar{G}^0 + 5G^0\right)^2 + \omega^2\left(\bar{G}^0\bar{\theta}_1^0 + 5G^0\theta_1^0\right)^2\right\}^{1/2}\right]^{1/2}, \end{aligned} \tag{4.6}$$

and

$$\begin{aligned} D_3(k) &= \frac{\theta_0\left(m_1^0\right)^2(1 - i\omega t_1)\rho_0\omega_1^2}{(a_1\gamma_0)^2 + \left(b_1\gamma_0 - \rho_0\eta_0\left(\omega_1^2/\omega^2\right)\right)^2\omega^2}\left\{a_1\gamma_0 + i\omega\left(b_1\gamma_0 - \rho_0\eta_0\frac{\omega_1^2}{\omega^2}\right)\right\} \\ &= \operatorname{Re} D_3(k_c) + \operatorname{Im} D_3(k_c), \end{aligned} \tag{4.7}$$

say, where

$$\lambda_0 + 2\mu_0 = a_1 - i\omega b_1,$$

and

$$a_1 = \frac{1}{3}\left(\bar{G}^0 + 5G^0\right), \qquad b_1 = \frac{1}{3}\left(\bar{G}^0\bar{\theta}_1^0 + 5G^0\theta_1^0\right). \tag{4.8}$$

Also, from (4.6) and (4.8), we can write

$$a_1 = \frac{(a^2 - b^2)}{3} \quad \text{and} \quad b_1 = -\frac{2ab}{2\omega}. \tag{4.9}$$

For the evaluation of the integral in (3.26), let us assume that

$$R_{mm}(r) = \left(m_1^0\right)e^{-a_0 r}, \qquad a_0 > 0. \tag{4.10}$$

Then $D_4(k_c)$ in (3.26) can be written as

$$D_4(k_c) = (1 - i\omega t_1)(1 + \delta_{lk}\omega t_0)\omega\theta_0 k_c\left(m_1^0\right)^2 \cdot \int_0^\infty \left(e^{-a_0 r} - 1\right)e^{(-\beta_1 + i\beta_2)r}\sin(k_c r)\,dr, \tag{4.11}$$

where

$$\begin{aligned} \beta_1 &= \sqrt{\frac{\omega}{\eta_0}}\left(\gamma_0^2 + \omega^2\delta_0^{*2}\right)^{1/4}\sin\frac{\theta'}{2}, \\ \beta_2 &= \sqrt{\frac{\omega}{\eta_0}}\left(\gamma_0^2 + \omega^2\delta_0^{*2}\right)^{1/4}\cos\frac{\theta'}{2}. \end{aligned} \tag{4.12}$$

After evaluation of the complicated integral in $D_4(k_c)$, let us write

$$D_4(k_c) = \operatorname{Re} D_4(k_c) + i\operatorname{Im} D_4(k_c), \tag{4.13}$$

where the complicated expressions for $\operatorname{Re} D_3(k_c)$, $\operatorname{Im} D_3(k_c)$, $\operatorname{Re} D_4(k_c)$, and $\operatorname{Im} D_4(k_c)$ are given in the Appendix.

From (4.1), we can now write

$$k = \left[\frac{\rho_0}{(\lambda_0 + 2\mu_0)}\right]^{1/2} \omega_1 + \frac{1}{2} \cdot \frac{\omega}{\omega_1} \cdot i \frac{\nu_0^2 \sigma_1^0 H_{02}^2}{\sqrt{\rho_0(\lambda_0 + 2\mu_0)}} \epsilon - \frac{[D_3(k_c) + D_4(k_c)]}{2\omega_1 [\rho_0(\lambda_0 + 2\mu_0)]^{1/2}} \epsilon^2$$
$$- \frac{2\rho_0 \omega^2 \Omega^2}{\omega_1^2 k_c (\lambda_0 + \mu_0)} \left[1 - \frac{i\epsilon\omega\nu_0^2\sigma_1^0}{\rho_0\omega_1^2(\lambda_0 + \mu_0)} \left\{(\lambda_0 + 2\mu_0) H_{01}^2 + \mu_0 H_{02}^2\right\} \right. \tag{4.14}$$
$$\left. + \frac{\mu_0 [D_3(k_c) + D_4(k_c)] \epsilon^2}{\rho_0 \omega_1^2 (\lambda_0 + \mu_0)}\right].$$

From (4.4), we can write

$$\lambda_0 + \mu_0 = (a_2 - i\omega b_2),$$

where

$$a_2 = \frac{1}{3}\left(\bar{G}^0 + 2G^0\right), \qquad b_2 = \frac{1}{3}\left(\bar{G}^0 \bar{\theta}_1^0 + 2G^0 \theta_1^0\right). \tag{4.15}$$

From (4.8) and (4.15), we can write

$$\mu_0 = a_3 - i\omega b_3,$$

where

$$a_3 = a_1 - a_2 \quad \text{and} \quad b_3 = b_1 - b_2. \tag{4.16}$$

Let us now write from (4.14),

$$k = k_{\rm I} + k_{\rm II}, \tag{4.17a}$$

where

$$k_{\rm I} = \frac{\sqrt{\rho_0}\omega_1 \sqrt{3}}{a^2 + b^2}(a - ib) + \frac{\sqrt{3}\,\omega \in \sigma^0 \nu_0^2 H_{02}^2 (b + ia)}{2\omega_1 \sqrt{\rho_0}(a^2 + b^2)} - \frac{(a - ib)\sqrt{3}\,\epsilon^2}{2\sqrt{\rho_0}\omega_1 (a^2 + b^2)}$$
$$\cdot \left\{\operatorname{Re} D_3(k_c) + i \operatorname{Im} D_3(k_c) + \operatorname{Re} D_4(k_c) + i \operatorname{Im} D_4(k_c)\right\}, \tag{4.17b}$$

$$k_{\rm II} = -\frac{2}{\sqrt{3}} \frac{\rho_0 \omega^2 \Omega^2 \{(aa_2 - \omega b b_2) + i(ba_2 + \omega a b_2)\}}{\omega_1^3 \sqrt{\rho_0}\,(a_2^2 + \omega^2 b_2^2)}$$
$$\cdot \left[1 - \frac{\epsilon\omega\sigma_1^0\nu_0^2(a_2 i - \omega b_2)}{\rho_0\omega_1^2\,(a_2^2 + \omega^2 b_2^2)} \cdot \left\{(a_1 - i\omega b_1) H_{01}^2 + (a_3 - i\omega b_3) H_{02}^2\right\}\right.$$
$$+ \frac{\epsilon^2 \left\{\left(a_2 a_3 + \omega^2 b_2 b_3\right) + i(a_3 b_2 - a_2 b_3)\omega\right\}}{\rho_0 \omega_1^2\,(a_2^2 + \omega^2 b_2^2)}$$
$$\left. \cdot \left\{\operatorname{Re} D_3(k_c) + i \operatorname{Im} D_3(k_c) + \operatorname{Re} D_4(k_c) + i \operatorname{Im} D_4(k_c)\right\}\right]. \tag{4.17c}$$

Then

$$\operatorname{Re} k = \operatorname{Re} k_{\rm I} + \operatorname{Re} k_{\rm II} \quad \text{and} \quad \operatorname{Im} k = \operatorname{Im} k_{\rm I} + \operatorname{Im} k_{\rm II}. \tag{4.18}$$

From the solution written above, it is quite clear that the effect of randomness to the ϵ-order term is to increase attenuation of longitudinal type waves, the attenuation coefficient being equal to

$$\left(\frac{\omega}{\omega_1 \sqrt{\rho_0}}\right) \sigma_1^0 \nu_0^2 \left[\frac{\sqrt{3}\,a}{a^2 + b^2} \frac{H_{02}^2}{2} + \frac{2}{\sqrt{3}} \cdot \frac{\omega^2 \Omega^2}{\omega_1^4 (a_2^2 + \omega^2 b_2^2)}\right.$$
$$\cdot \left\{\left[(aa_2 - \omega b b_2)\left(a_1 a_2 + b_1 b_2 \omega^2\right) + (ba_2 + \omega a b_2)\left(b_1 a^2 - a_1 b_2\right)\omega\right] H_{01}^2\right. \tag{4.19}$$
$$\left.\left. + \left[(aa_2 - \omega b b_2)\left(a_2 a_3 + \omega^2 b_2 b_3\right) + (ba_2 + \omega a b_2)(b_3 a_2 - a_3 b_2)\omega\right] H_{02}^2\right\}\right],$$

in the Voigt-type visco-elastic medium interacting with a weak magneto-thermal field. Also, the phase velocity increases at the rate of

$$\begin{aligned}&\left(\frac{\omega}{\omega_1\sqrt{\rho_0}}\right)\sigma_1^0\nu_0^2\left[\frac{\sqrt{3}\,a}{a^2+b^2}\frac{H_{02}^2}{2}+\frac{2}{\sqrt{3}}\cdot\frac{\omega^2\Omega^2}{\omega_1^4(a_2^2+\omega^2 b_2^2)}\right.\\&\cdot\left\{\left[(aa_2-\omega bb_2)\left(a_1a_2+b_1b_2\omega^2\right)+(ba_2+\omega ab_2)\left(b_1a^2-a_1b_2\right)\omega\right]H_{01}^2\right.\\&\left.\left.+\ \left[(aa_2-\omega bb_2)\left(a_2a_3+\omega^2b_2b_3\right)+(ba_2+\omega ab_2)(b_3a_2-a_3b_2)\omega\right]H_{02}^2\right\}\right],\end{aligned} \tag{4.20}$$

to the ϵ-order term, and the increase depends upon the visco-elastic parameters. The ϵ-order effect is absent in the absence of the magnetic field. The effect of rotation can be easily seen through the presence of Ω in each term. However, the effect of generalised thermoelasticity is, also, not discernible in this case as can be seen from the absence of t_0 and t_1 in the above terms. The effect of thermoelastic interaction, however, is discernible in the ϵ^2-order terms only. The contributions of the terms $D_3(k_c)$ and $D_4(k_c)$ are analysed below.

Low Frequency Waves

For small ω, the ϵ^2 term due to $D_3(k_c)$ in $\operatorname{Im} k$ of equation (4.18) along with (4.19) and (4.20) approximates to

$$\frac{\sqrt{\rho_0\theta_0}\left(m_1^0\right)^2\omega\omega_1\left[a_1(2\rho_0\eta_0-3b_1\gamma_0)+t_1\left(2a_1^2\gamma_0+\rho_0\eta_0b_1\Omega^2\right)\right]}{4a_1^{3/2}\left[(a_1\gamma_0)^2+2\rho_0\eta_0\Omega^2(\rho_0\eta_0-b_1\gamma_0)\right]}. \tag{4.21}$$

This shows that if η_0 increases, attenuation also increases provided

$$\eta_0\rho_0>\frac{3}{2}b_1\gamma_0. \tag{4.22}$$

Hence, it is observed that attenuation increases with the increase of thermal conductivity along with rotation and thermal relaxation parameter for low frequency waves.

Otherwise, the effect of visco-elastic parameters a_1, b_1 and the rotation parameter Ω involved in the analysis is to diminish attenuation. However, attenuation increases proportionately to the square of the mean of the thermoelastic coupling parameter in this case.

Again, the ϵ^2-term due to $D_3(k_c)$ in $\operatorname{Re} k$ approximates to

$$-\frac{\sqrt{\rho_0\theta_0}\left(m_1^0\right)^2\omega_1}{4a_1^{3/2}}\frac{2a_1^2\gamma_0+\omega^2\{(b_1\gamma_0-2\rho_0\eta_0)a_1t_1+b_1(b_1\gamma_0-\rho_0\eta_0)\}}{(a_1\gamma_0)^2+2\rho_0\eta_0\Omega^2(\rho_0\eta_0-b_1\gamma_0)}. \tag{4.23}$$

The expression (4.23) is independent of t_0 but depends on t_1 and rotation parameter Ω, if we keep terms up to the order of ω and it indicates the increase of phase velocity provided, of course, if the condition $\rho_0\eta_0 > b_1\gamma_0$ is satisfied. If $\Omega \to 0$, then the expression (4.23) will exactly coincide with the result obtained in [21]. Furthermore, although the effect of relaxation parameter (t_0) is not perceptible up to the order of approximation unless it is increased, yet the effect of t_1 and rotation parameter Ω is appreciable.

The ϵ^2 terms due to $D_4(k_c)$ in $\operatorname{Im} k$ and $\operatorname{Re} k$ approximate, respectively, to

$$\frac{\sqrt{\rho_0\theta_0}\left(m_1^0\right)^2\eta_0}{2(a_1)^{7/2}\gamma_0^2}\omega\omega_1\left[\frac{a_1b_1\gamma_0}{\eta_0}+\frac{\rho_0\Omega^2}{a_1}\left(b_1^2+a_1^2t_0t_1\delta_{lk}\right)-\left\{(t_1+t_0\delta_{lk})\left(\frac{a_1^2\gamma_0}{\eta_0}+3\rho_0b_1\Omega^2\right)\right\}\right] \tag{4.24}$$

and

$$\frac{\sqrt{\rho_0\theta_0}\left(m_1^0\right)^2\eta_0}{4(a_1)^{9/2}\gamma_0^2}\omega_1\left[\rho_0\Omega^2a_1\{b_1-2a_1(t_1+t_0\delta_{lk})\}\right]. \tag{4.25}$$

Thus, it is observed that the attenuation increases on account of the term $D_4(k_c)$, if the term up to the order of ω^2 is considered, and we have to assume that

$$\left[\frac{b_1\gamma_0}{\eta_0} + \frac{\rho_0\Omega^2}{a_1}\left(b_1^2 + t_0 t_1 \delta_{lk} a_1^2\right)\right] > \left[\left(3\rho_0 b_1 \Omega^2 + \frac{a_1^2\gamma_0}{\eta_0}\right)(t_1 + t_0\delta_{lk})\right]. \tag{4.26}$$

Otherwise, the attenuation will decrease in this case. So, the contribution of $D_4(k_c)$ to increase in attenuation is more important than that of $D_3(k_c)$. Also, phase velocity increases with the increase of η_0, if the term up to the order of ω is considered and we have to assume that

$$b_1 > 2a_1(t_1 + t_0\delta_{lk}). \tag{4.27}$$

Otherwise, the velocity decreases. In all these cases, there is significant contribution of t_0, t_1, and Ω.

High Frequency Waves

For large ω, $\theta' \to 0$ and $(\beta_1, \beta_2) \to (0, \omega\sqrt{\delta_0^*/\eta_0})$. Then the ϵ^2-term due to $D_3(k_c)$ in $\operatorname{Im} k$ and $\operatorname{Re} k$ are, respectively,

$$\begin{aligned} -\frac{\sqrt{\rho_0}\,\theta_0\left(m_1^0\right)^2\epsilon^2}{2\sqrt{2b'}(b_1\gamma_0 - \rho_0\eta_0)^2}\cdot\Bigg[\Big\{-a_1\gamma_0 t_1 + (b_1\gamma_0 - \rho_0\eta_0)\Big\}\frac{1}{\sqrt{\omega}} \\ + t_1(b_1\gamma_0 - \rho_0\eta_0)\left(\omega^{1/2} + \frac{\Omega^2}{\omega^{3/2}}\right)\Bigg] \end{aligned} \tag{4.28a}$$

and

$$-\frac{\sqrt{\rho_0}\,\theta_0\left(m_1^0\right)^2\epsilon^2}{2\sqrt{2b'}(b_1\gamma_0-\rho_0\eta_0)^2}\left[t_1(b_1\gamma_0 - \rho_0\eta_0)\left(\sqrt{\omega} + \frac{\Omega^2}{\omega^{3/2}}\right) + \{a_1\gamma_0 t_1 - (b_1\gamma_0 - \rho_0\eta_0)\}\frac{1}{\sqrt{\omega}}\right]. \tag{4.28b}$$

The attenuation diminishes with increase of thermal conductivity. However, attenuation increases proportionately to $(m_1^0)^2$ as before but for

$$b_1\gamma_0 > \rho_0\eta_0 + a_1\gamma_0 t_1.$$

Phase velocity behaves in a similar manner as that of the attenuation. It is also to be noted that to study the effect of rotation, the order of ω is considered up to $\omega^{-3/2}$. If $(t_0, t_1, \Omega) \to 0$, we can identify the corresponding results obtained in [6]. If $\Omega \to 0$, we can get back the results obtained in [21].

The ϵ^2-terms due to $D_4(k_c)$ in $\operatorname{Im} k$ and $\operatorname{Re} k$ are, respectively, obtained as

$$\begin{aligned} -\frac{\epsilon^2\sqrt{\rho_0}\,\theta_0\left(m_1^0\right)^2\eta_0^2}{2\sqrt{2b_1}\cdot(b_1\delta_0^*)^2}\Bigg[2(t_1 + t_0\delta_{lk})\frac{b_1\delta_0^*}{\eta_0} + \frac{1}{\omega}(t_1 + t_0\delta_{lk}) \\ \cdot\left(\rho_0 + 2a_0 b_1\sqrt{\frac{\delta_0^*}{\eta_0}} - a_1\frac{\delta_0^*}{\eta_0}\right) + \frac{1}{\omega^2}\Bigg\{[1 - t_0 t_1(1+\Omega^2)\delta_{lk}] \\ \cdot\left(\rho_0 + 2a_0 b_1\sqrt{\frac{\delta_0^*}{\eta_0}} - a_1\frac{\delta_0^*}{\eta_0}\right) + 2\Omega^2(t_1 + t_0\delta_{lk})\left(\frac{b_1\delta_0}{\eta_0}\right)\Bigg\}\Bigg] \end{aligned} \tag{4.29a}$$

and

$$\begin{aligned} -\frac{\epsilon^2\sqrt{\rho_0}\,\theta_0\left(m_1^0\right)^2\eta_0^2}{2\sqrt{2b_1}\cdot(b_1\delta_0^*)^2}\Bigg[2b_1\frac{\delta_0^*}{\eta_0}(t_1 + t_0\delta_{lk}) - \frac{1}{\omega}(t_1 + t_0\delta_{lk}) \\ \cdot\left(\rho_0 + 2a_0 b_1\sqrt{\frac{\delta_0^*}{\eta_0}} - a_1\frac{\delta_0^*}{\eta_0}\right) + \frac{1}{\omega^2}\Bigg\{[1 - (1+\Omega^2)t_0 t_1\delta_{lk}] \\ \cdot\left(\rho_0 + 2a_0 b_1\sqrt{\frac{\delta_0^*}{\eta_0}} - a_1\frac{\delta_0^*}{\eta_0}\right) + 2\Omega^2\left(\frac{b_1\delta_0}{\eta_0}\right)(t_1 + t_0\delta_{lk})\Bigg\}\Bigg]. \end{aligned} \tag{4.29b}$$

The attenuation increases with the increase in thermal conductivity. This result is different from that obtained in the case of $D_3(k_c)$. Phase velocity increases in this case also. However, attenuation and phase velocity increase proportionately to the square of the mean of the thermoelastic coupling parameter (m_1^0). It is also to be noted that to study the rotational effect, the order of ω is considered up to ω^{-2}. If $t_0 \to 0$ and if $(t_1, \Omega) \to 0$, the results in the present analysis can be identified with those of [6] and [21]. But the results obtained in [6] require certain correction before verification.

It can similarly be shown that the thermal field does not affect propagation of transverse type waves represented by $k = k_s - i\delta_s$, $k_s^2 = \rho_0\omega^2/\mu_0$, even to the order of ϵ^2.

The case of Maxwell-type visco-elastic solids can be similarly treated.

5. NUMERICAL RESULTS AND DISCUSSIONS

The numerical computation of phase velocity and attenuation coefficient for Low Frequency Waves for the visco-elastic material like polythene (I.C.I. Alkathene-grade 20) [23] for $\nu = 0.25$ at 12°K has been done for small values of m_1^0 with the help of PC, and the corresponding graphs have been drawn. The units of different dimensionless parameters have been chosen to suit the purpose of the present analysis.

Taking

$$
\begin{aligned}
\rho_0 &= 0.92\,\mathrm{Kg\,m^{-3}}, & \theta_0 &= 12\,\mathrm{K}, \\
\gamma_0 &= 2.3184\,\mathrm{JK^{-1}\,m^{-3}}, & \eta_0 &= 0.19\,\mathrm{Js^{-1}\,m^{-1}K^{-1}}, \\
m_1^0 &= 0.5\,\mathrm{Nm^{-2}K^{-1}}, & t_0 &= 10^{-5}\,\mathrm{s}, \\
t_1 &= 10^{-5}\,\mathrm{s}, & \omega &= 10^6\,\mathrm{s^{-1}}, \\
\Omega &= 10^5\,\mathrm{s^{-1}}, & G_0 &= 1.7\times 10^9\,\mathrm{Nm^{-2}}, \\
a_1 &= 2.5\,G^0, & b_1 &= 0.0425\,\mathrm{Nm^{-2}s},
\end{aligned}
$$

for the material under consideration, the following graphs have been drawn under G-L theory, as an illustration to show the effect of thermal relaxation (τ_0, τ_1) and of rotation (Ω) on *attenuation* and *phase velocity* corresponding to low frequency. The graphs are also drawn for different values of m_1^0 under the effect of rotation and thermal relaxation.

Figures 1–4 illustrate the graphs of attenuation, and Figures 5–8 illustrate the graphs of phase velocity for different values of the dimensionless parameter x_i $(= \rho_0\eta_0/b_1\gamma_0)$ and frequency ω. Figure 9 illustrates the graph of attenuation for $\tau_1 = \tau_0 = 0$, that is, under classical dynamic coupled theory.

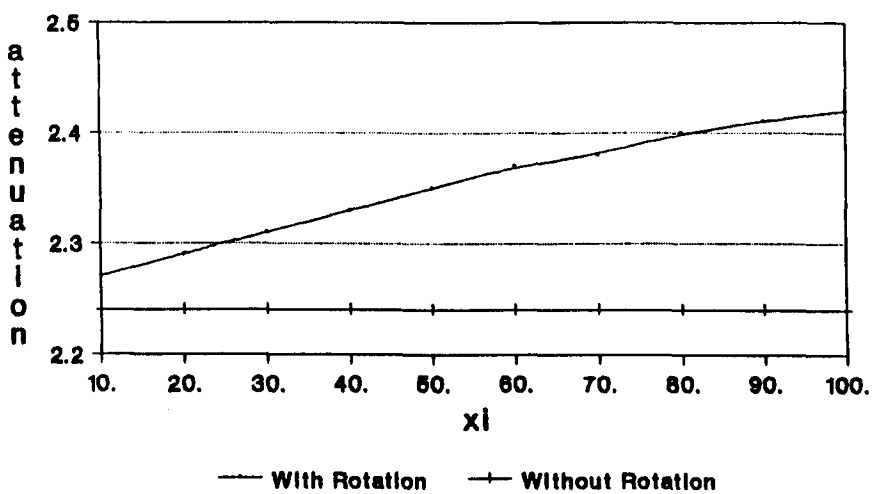

Figure 1. Attenuation for different values of x_i.

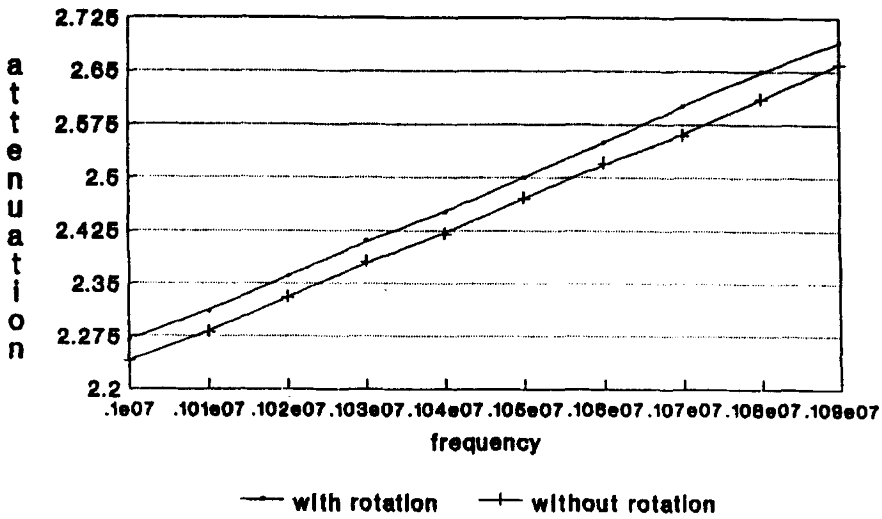

Figure 2. Attenuation vs. frequency.

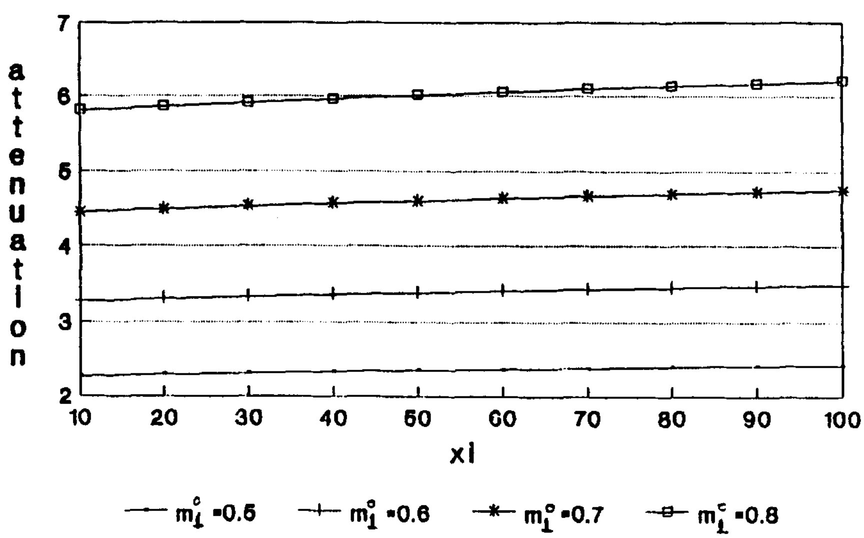

Figure 3. Attenuation for different m_1^0.

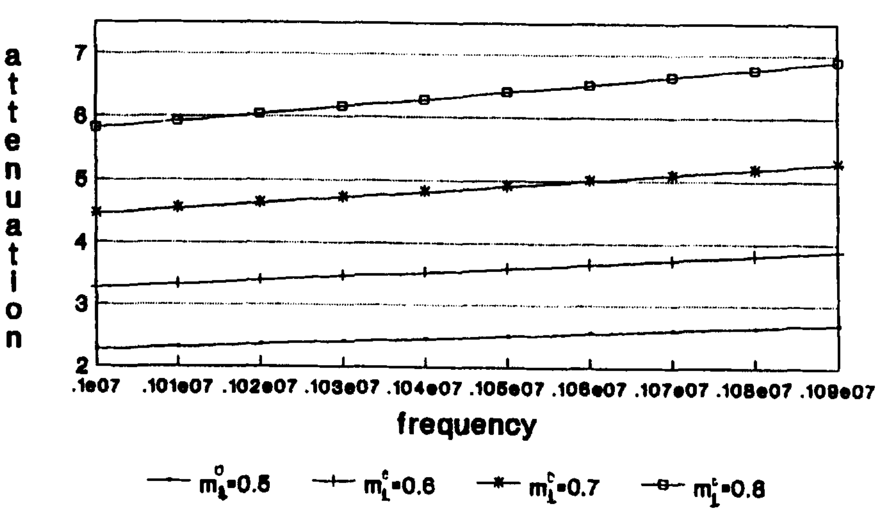

Figure 4. Attenuation vs. frequency for different m_1^0.

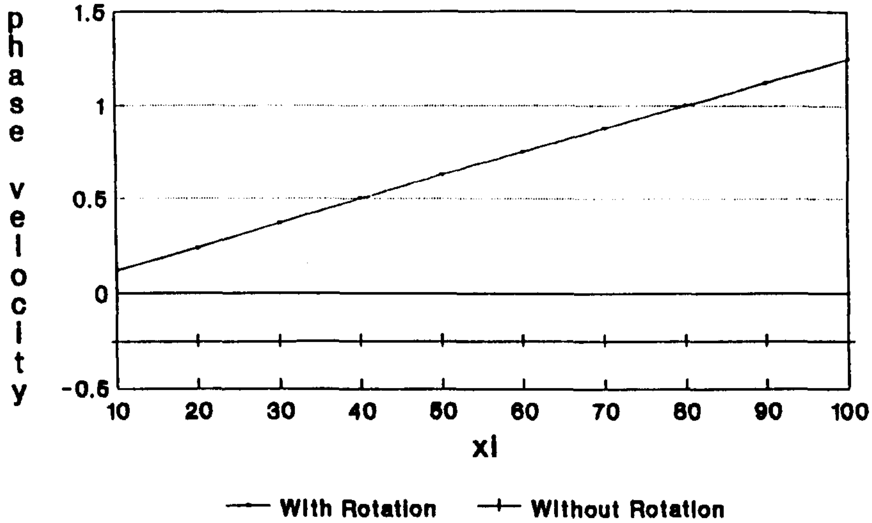

Figure 5. Phase velocity vs. x_i.

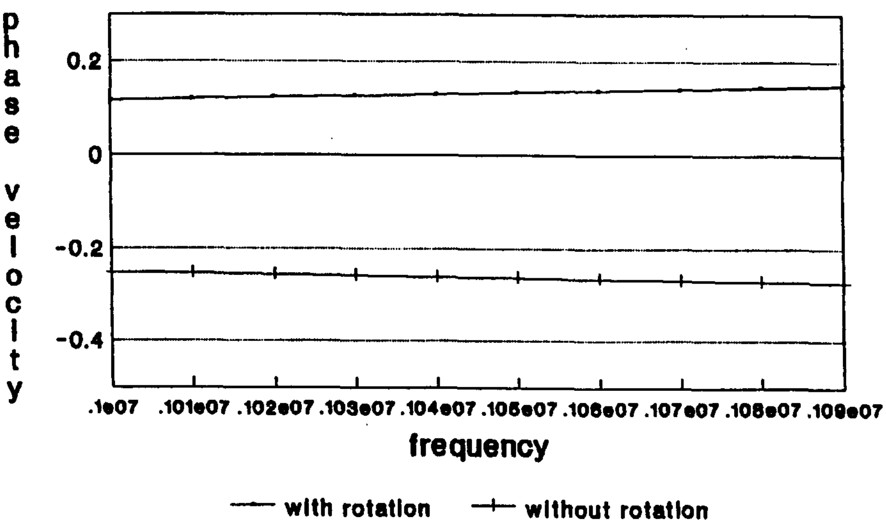

Figure 6. Phase velocity vs. frequency.

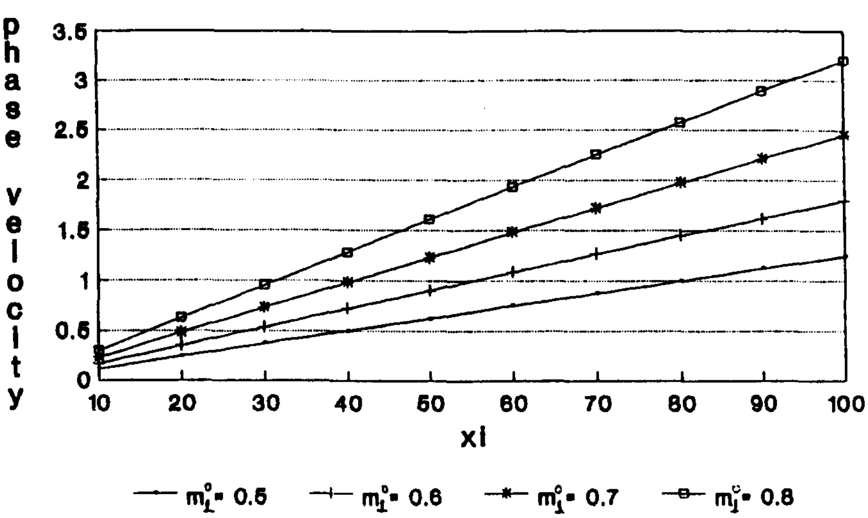

Figure 7. Phase velocity vs. x_i for different values of m_1^0.

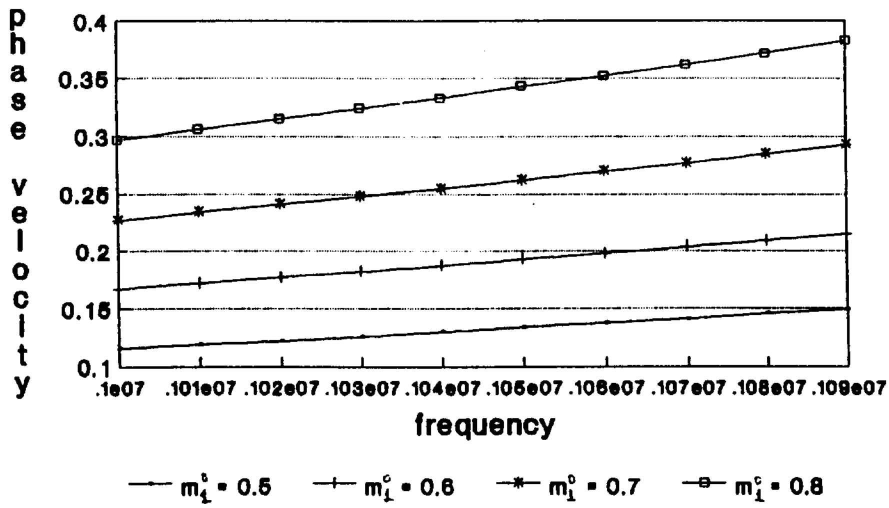

Figure 8. Phase velocity vs. frequency for different values of m_1^0.

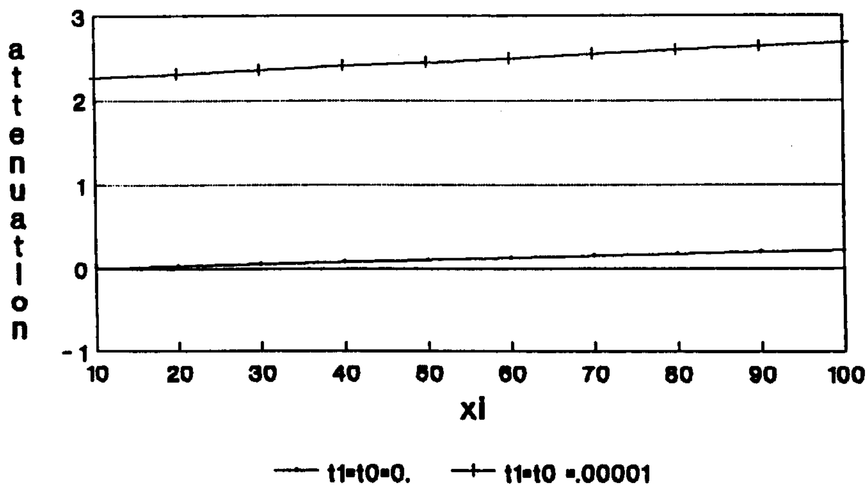

Figure 9. Attenuation for $t_1 = t_0 = 0$ and $t_1 = t_0 = 0.00001$.

6. CONCLUSION

General thermo-mechanical coupling could not be considered as the problem becomes intractable. That is why the case of weak coupling has been discussed (cf. [4]). Furthermore, in the present problem, only the thermal correlation has been taken as nonzero so as to get a numerically tractable solution. However, taking similar procedure, numerical computation for other nonvanishing correlation functions can be taken up. Some illustrative graphs have been drawn to show the effect of rotation in generalised theory and in classical theory in random media. The graphs for the case of high frequency can be similarly drawn under the above-mentioned effects.

APPENDIX

I. Real and Imaginary Parts of $D_3(k_c)$:

$$\operatorname{Re} D_3(k_c) = \rho_0 \omega \omega_1^2 \left(m_1^0\right)^2 \frac{\left\{\omega a_1 \gamma_0 + \omega t_1 \left(b_1 \gamma_0 \omega^2 - \rho_0 \eta_0 \omega_1^2\right)\right\}}{(a_1\gamma_0)^2 \omega^2 + \left(b_1 \gamma_0 \omega^2 - \rho_0 \eta_0 \omega_1^2\right)^2},$$

$$\operatorname{Im} D_3(k_c) = -\rho_0 \omega \omega_1^2 \left(m_1^0\right)^2 \frac{\left\{a_1 \gamma_0 t_1 \omega^2 - \left(b_1 \gamma_0 \omega^2 - \rho_0 \eta_0 \omega_1^2\right)\right\}}{(a_1\gamma_0)^2 \omega^2 + \left(b_1 \gamma_0 \omega^2 - \rho_0 \eta_0 \omega_1^2\right)^2}.$$

II. Real and Imaginary Parts of $D_4(k_c)$:

$$\begin{aligned}
\operatorname{Re} D_4(k_c) &= n_0 \left[\omega(t_1 + t_0\delta_{lk}) \left\{ \frac{n_2n_3 + n_1n_4}{n_2^2 + n_4^2} - \frac{n_5n_6 - n_7n_8}{n_6^2 + n_8^2} \right\} \right. \\
&\quad \left. + \left(1 - \omega^2 t_0 t_1 \delta_{lk}\right) \left\{ \frac{n_2n_3 - n_1n_4}{n_2^2 + n_4^2} + \frac{n_5n_6 + n_7n_8}{n_6^2 + n_8^2} \right\} \right], \\
\operatorname{Im} D_4(k_c) &= n_0 \left[\left(1 - \omega^2 t_0 + t_1\delta_{lk}\right) \left\{ \frac{n_2n_3 + n_1n_4}{n_2^2 + n_4^2} - \frac{n_5n_6 - n_7n_8}{n_6^2 + n_8^2} \right\} \right. \\
&\quad \left. + \omega(t_1 t_0 \delta_{lk}) \left\{ \frac{n_2n_3 - n_1n_4}{n_2^2 + n_4^2} + \frac{n_5n_6 + n_7n_8}{n_6^2 + n_8^2} \right\} \right],
\end{aligned}$$

where

$$\begin{aligned}
n_0 &= \rho_0\omega\theta_0 \left(m_1^0\right)^2 \omega_1^2, \\
n_1 &= \omega a_0 b'(a_0 + 2\beta_1) - 2a'\beta_2(a_0 + \beta_1) + \omega b' \left(\beta_1^2 - \beta_2^2\right), \\
n_2 &= \left(\rho_0\omega^2\right)^2 + 2a'\rho_0\omega_1^2 \left\{ a_0 \left(a_0^2 + 2\beta_1\right) + \left(\beta_1^2 - \beta_2^2\right) + \left(a'^2 - \omega^2 b'^2\right) \left\{ (a_0 + \beta_1)^2 + \beta_2 \right\}^2, \right. \\
n_3 &= \rho_0\omega_1^2 + a_0 a'(a_0 + 2\beta_1) + 2\beta_2 b'\omega(a_0 + \beta_1) + a_1 \left(\beta_1^2 - \beta_2^2\right), \\
n_4 &= 2\rho_0\omega_1^2\omega b' \left\{ a_0(a_0 + 2\beta_1) + \beta_1^2 - \beta_2^2 \right\} + 2a'b'\omega \left\{ (a_0 + \beta_1)^2 + \beta_2^2 \right\}^2, \\
n_5 &= \rho_0\omega_1^2 + 2\omega b'\beta_1^2 + a_1 \left(\beta_1^2 - \beta_2^2\right), \\
n_6 &= \left(\rho_0\omega_1^2\right)^2 + \left(\beta_1^2 + \beta_2^2\right)^2 \left(a'^2 - \omega^2 b'^2\right) + 2\rho_0\omega_1^2 a' \left(\beta_1^2 - \beta_2^2\right), \\
n_7 &= 2a_1\beta_1\beta_2 - b_1\omega \left(\beta_1^2 - \beta_2^2\right), \\
n_8 &= 2 \left[a_1 b_1 \omega \left(\beta_1^2 + \beta_2^2\right) + \rho_0 b_1 \omega\omega_1^2 \left(\beta_1^2 - \beta_2^2\right) \right],
\end{aligned}$$

where $\beta_1 = \operatorname{Re}\beta$ and $\beta_2 = \operatorname{Im}\beta$.

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